Series JMS/1 **SET-1**

कोड नं.
Code No. **30/1/1**

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **30** प्रश्न हैं।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **30** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*
Time allowed : 3 hours — *Maximum Marks : 80*

सामान्य निर्देश:

(i) सभी प्रश्न अनिवार्य हैं।

*(ii) इस प्रश्न-पत्र में **30** प्रश्न हैं जो चार खण्डों – अ, ब, स और द में विभाजित हैं।*

*(iii) खण्ड अ में एक-एक अंक वाले **6** प्रश्न हैं। खण्ड ब में **6** प्रश्न हैं जिनमें से प्रत्येक **2** अंक का है। खण्ड स में **10** प्रश्न तीन-तीन अंकों के हैं। खण्ड द में **8** प्रश्न हैं जिनमें से प्रत्येक **4** अंक का है।*

(iv) प्रश्न-पत्र में कोई समग्र विकल्प नहीं है। तथापि एक अंक वाले दो प्रश्नों में, दो अंकों वाले 2 प्रश्नों में, 3 अंकों वाले 4 प्रश्नों में और चार अंकों वाले 3 प्रश्नों में आंतरिक विकल्प प्रदान किए गए हैं। ऐसे प्रश्नों में आपको दिए गए विकल्पों में से केवल एक प्रश्न ही करना है।

(v) कैलकुलेटर के प्रयोग की अनुमति नहीं है।

General Instructions :

(i) **All** questions are compulsory.

(ii) This question paper consists of **30** questions divided into four sections – **A, B, C** and **D**.

(iii) Section **A** contains **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of **4** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternative in all such questions.

(v) Use of calculator is **not** permitted.

खण्ड – अ

SECTION - A

प्रश्न संख्या **1** से **6** तक प्रत्येक प्रश्न 1 अंक का है।

Question numbers **1** to **6** carry 1 mark each.

1. एक वृत्त जिसका केन्द्र (2, –3) है, का एक व्यास AB है। यदि बिंदु B के निर्देशांक (1, 4) हैं तो बिंदु A के निर्देशांक ज्ञात कीजिए।

Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4).

2. k के किन मानों के लिए समीकरण $x^2 + 4x + k = 0$ के मूल वास्तविक होंगे ?

अथवा

k का वह मान ज्ञात कीजिए जिसके लिए समीकरण $3x^2 - 10x + k = 0$ के मूल एक-दूसरे के प्रतिलोम हों।

For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

Or

Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

3. यदि tan 2A = cot (A –24°) है, तो A का मान ज्ञात कीजिए।

अथवा

$(\sin^2 33^\circ + \sin^2 57^\circ)$ का मान ज्ञात कीजिए।

Find A if tan 2A = cot (A –24°)

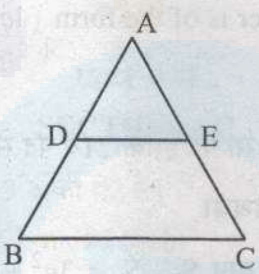
Or

Find the value of $(\sin^2 33^\circ + \sin^2 57^\circ)$

4. दो अंकों की कितनी संख्याएँ 3 से भाज्य हैं ?

How many two digits numbers are divisible by 3 ?

5. आकृति 1 में, DE || BC, AD = 1 सेमी तथा BD = 2 सेमी है। (Δ ABC) तथा (Δ ADE) के क्षेत्रफलों में क्या अनुपात है ?



आकृति 1

In Fig. 1, DE || BC, AD = 1 cm and BD = 2 cm. What is the ratio of the ar (Δ ABC) to the ar (Δ ADE) ?

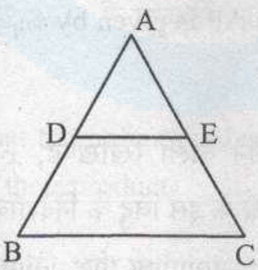


Fig. 1

6. $\sqrt{2}$ तथा $\sqrt{3}$ के बीच में स्थित एक परिमेय संख्या ज्ञात कीजिए।

Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

खण्ड – ब

SECTION – B

प्रश्न संख्या **7** से **12** तक प्रत्येक प्रश्न 2 अंकों का है।

Question numbers **7** to **12** carry 2 marks each.

7. यूक्लिड एल्गोरिथ्म के प्रयोग से 1260 तथा 7344 का महत्तम समापवर्तक (HCF) ज्ञात कीजिए।

अथवा

दर्शाइए कि प्रत्येक विषम धनपूर्णांक $(4q+1)$ अथवा $(4q+3)$ के रूप का होता है, जहाँ q कोई पूर्णांक है।

Find the HCF of 1260 and 7344 using Euclid's algorithm.

Or

Show that every positive odd integer is of the form $(4q+1)$ or $(4q+3)$, where q is some integer.

8. समांतर श्रेढ़ी 3, 15, 27, 39, ... का कौन सा पद इसके 21वें पद से 120 अधिक है ?

अथवा

यदि एक समांतर श्रेढ़ी के प्रथम n पदों का योग S_n, $S_n = 3n^2 - 4n$ द्वारा प्रदत्त है, तो इसका nवाँ पद ज्ञात कीजिए।

Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term ?

Or

If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term.

9. बिंदुओं (1, – 3) तथा (4, 5) को मिलाने वाला रेखाखण्ड, x-अक्ष द्वारा जिस अनुपात में विभाजित होता है, वह ज्ञात कीजिए। x-अक्ष के इस बिंदु के निर्देशांक भी ज्ञात कीजिए।

Find the ratio in which the segment joining the points (1, – 3) and (4, 5) is divided by x-axis ? Also find the coordinates of this point on x-axis.

10. एक खेल में एक रुपए के सिक्के को तीन बार उछाला जाता है और प्रत्येक बार का परिणाम लिख लिया जाता है। यदि तीनों परिणाम समान होने को जीत माना जाए तो खेल हारने की प्रायिकता ज्ञात कीजिए।

A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

11. एक पासे को एक बार उछाला गया (i) एक अभाज्य संख्या के आने की (ii) 2 तथा 6 के बीच की संख्या के आने की, प्रायिकता ज्ञात कीजिए।

A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

12. c का मान ज्ञात कीजिए, यदि समीकरण निकाय $cx+3y+(3-c)=0; 12x+cy-c=0$ के अपरिमित रूप से अनेक हल हैं।

Find c if the system of equations $cx+3y+(3-c)=0; 12x+cy-c=0$ has infinitely many solutions ?

खण्ड – स

SECTION – C

प्रश्न संख्या **13** से **22** तक प्रत्येक प्रश्न 3 अंकों का है।

Question numbers **13** to **22** carry 3 marks each.

13. सिद्ध कीजिए कि $\sqrt{2}$ एक अपरिमेय संख्या है।

Prove that $\sqrt{2}$ is an irrational number.

14. यदि बहुपद $x^2-(k+6)x+2(2k-1)$ के शून्यकों का योग उनके गुणनफल का आधा है, तो k का मान ज्ञात कीजिए।

Find the value of k such that the polynomial $x^2-(k+6)x+2(2k-1)$ has sum of its zeros equal to half of their product.

15. एक पिता की आयु अपने दो बच्चों की आयु के योग के तीन गुने के समान है। 5 वर्ष के पश्चात उसकी आयु बच्चों की आयु के योग के दुगुने के समान होगी। पिता की वर्तमान आयु ज्ञात कीजिए।

अथवा

एक भिन्न $\frac{1}{3}$ हो जाती है, जब उसके अंश से 2 घटाया जाता है, और वह $\frac{1}{2}$ हो जाती है, जब हर में से 1 घटाया जाए। वह भिन्न ज्ञात कीजिए।

A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Or

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

16. y-अक्ष का वह बिंदु ज्ञात कीजिए जो बिंदुओं (5, –2) तथा (–3, 2) से समदूरस्थ है।

अथवा

बिंदुओं A(2, 1) तथा B(5, –8) को मिलाने वाला रेखाखण्ड बिंदुओं P तथा Q पर समत्रिभाजित होता है जबकि P बिंदु A के निकट है। यदि P, $2x-y+k=0$ द्वारा प्रदत्त रेखा पर भी स्थित है, तो k का मान ज्ञात कीजिए।

Find the point on y-axis which is equidistant from the points (5, –2) and (–3, 2).

Or

The line segment joining the points A(2, 1) and B(5, –8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x-y+k=0$, find the value of k.

17. सिद्ध कीजिए कि $(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta$.

अथवा

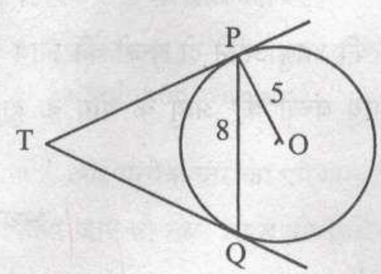
सिद्ध कीजिए कि $(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)=2$

Prove that $(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta$.

Or

Prove that $(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)=2$

18. आकृति 2 में, O केंद्र वाले 5 सेमी त्रिज्या के एक वृत्त की 8 सेमी लम्बी एक जीवा PQ है। P और Q पर स्पर्श रेखाएँ परस्पर एक बिंदु T पर प्रतिच्छेद करती हैं। TP की लंबाई ज्ञात कीजिए।



आकृति 2

In Fig. 2, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.

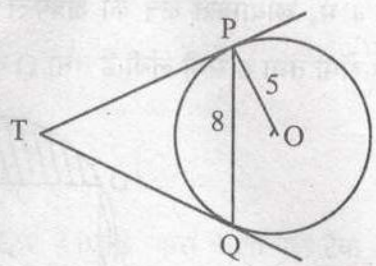
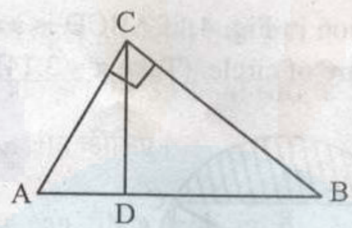


Fig. 2

19. आकृति 3 में, $\angle ACB = 90^\circ$ तथा $CD \perp AB$ है, सिद्ध कीजिए कि $CD^2 = BD \times AD$.



आकृति 3

अथवा

यदि P तथा Q क्रमशः Δ ABC की भुजाओं CA तथा CB पर स्थित बिंदु हैं तथा $\angle C$ समकोण है, तो सिद्ध कीजिए कि $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

In Fig. 3, $\angle ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.

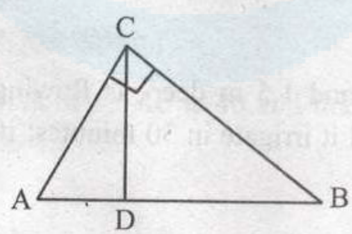
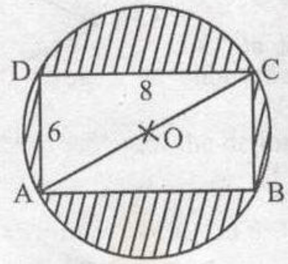


Fig. 3

Or

If P and Q are the points on side CA and CB respectively of Δ ABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

20. आकृति 4 में, छायांकित क्षेत्र का क्षेत्रफल ज्ञात कीजिए, यदि ABCD एक आयत है जिसकी भुजाएँ 8 सेमी तथा 6 सेमी लंबी हैं तथा O वृत्त का केंद्र है। ($\pi = 3.14$ लीजिए)



आकृति 4

Find the area of the shaded region in Fig. 4, if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take $\pi = 3.14$)

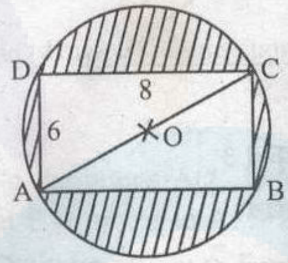


Fig. 4

21. 6 मी चौड़ी और 1.5 मी गहरी एक नहर में पानी 10 किमी/घंटा की चाल से बह रहा है। 30 मिनट में, यह नहर कितने क्षेत्रफल की सिंचाई कर पाएगी, जबकि सिंचाई के लिए 8 सेमी गहरे पानी की आवश्यकता होती है।

Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed ?

22. निम्न बारंबारता बंटन का बहुलक ज्ञात कीजिए।

वर्ग	0-10	10-20	20-30	30-40	40-50	50-60	60-70
बारंबारता	8	10	10	16	12	6	7

Find the mode of the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	10	16	12	6	7

खण्ड – द

SECTION – D

प्रश्न संख्या **23** से **30** तक प्रत्येक प्रश्न 4 अंकों का है।

Question numbers **23** to **30** carry 4 marks each.

23. दो पानी के नल एक साथ एक टैंक को $1\frac{7}{8}$ घंटों में भर सकते हैं। बड़े व्यास वाला नल टैंक को भरने में, कम व्यास वाले नल से 2 घंटे कम समय लेता है। प्रत्येक नल द्वारा अलग से टैंक को भरने का समय ज्ञात कीजिए।

अथवा

एक नाव 10 घंटे में धारा के प्रतिकूल 30 किमी तथा धारा के अनुकूल 44 किमी जाती है। 13 घंटे में वह 40 किमी धारा के प्रतिकूल एवं 55 किमी धारा के अनुकूल जाती है। धारा की चाल तथा नाव की स्थिर जल में चाल ज्ञात कीजिए।

Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Or

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

24. यदि एक समांतर श्रेढ़ी के प्रथम चार पदों का योग 40 है तथा प्रथम 14 पदों का योग 280 है। इस श्रेढ़ी के प्रथम n पदों का योग ज्ञात कीजिए।

If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

25. सिद्ध कीजिए $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

Prove that $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

26. 100 मी ऊँचे एक लाइट हाउस से दूर एक नाव को ले जाता हुआ व्यक्ति 2 मिनट में लाइट हाउस के शिखर के उन्नयन कोण को 60° से 30° बदलता हुआ पाता है। मीटर प्रति मिनट में नाव की चाल ज्ञात कीजिए। $\left[\sqrt{3} = 1.732 \text{ लीजिए}\right]$

अथवा

एक 80 मी चौड़ी सड़क के दोनों ओर आमने-सामने समान ऊँचाई वाले दो खंभे लगे हुए हैं। इन दोनों खंभों के बीच सड़क के एक बिंदु से खंभों के शिखर के उन्नयन कोण क्रमशः 60° और 30° हैं। खंभों की ऊँचाई और खंभों से बिंदु की दूरी ज्ञात कीजिए।

A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. $\left[\text{Use } \sqrt{3} = 1.732\right]$

Or

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

27. एक त्रिभुज ABC की रचना कीजिए जिसमें CA = 6 सेमी, AB = 5 सेमी तथा $\angle BAC = 45°$ हों। अब एक अन्य त्रिभुज की रचना कीजिए जिसकी भुजाएँ ΔABC की संगत भुजाओं का $\frac{3}{5}$ गुना हो।

Construct a Δ ABC in which CA = 6 cm, AB = 5 cm and $\angle$ BAC = 45°. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of Δ ABC.

28. शंकु के छिन्नक के आकार की ऊपर से खुली एक बाल्टी का आयतन 12308.8 घन सेमी है। इसके ऊपरी तथा निचले वृत्तीय सिरों की त्रिज्याएँ क्रमशः 20 सेमी तथा 12 सेमी हैं। बाल्टी की ऊँचाई तथा इसके बनाने में लगी धातु की चादर का क्षेत्रफल ज्ञात कीजिए। ($\pi = 3.14$ लीजिए)

A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use $\pi = 3.14$)

29. सिद्ध कीजिए कि एक समकोण त्रिभुज में कर्ण का वर्ग, अन्य दो भुजाओं के वर्गों के योग के समान होता है।

Prove that in a right angle triangle, the square of the hypotenuse is equal the sum of squares of the other two sides.

30. यदि निम्न बारंबारता बंटन का माध्यक 32.5 है तो f_1 तथा f_2 के मान ज्ञात कीजिए।

वर्ग :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	योग
बारंबारता :	f_1	5	9	12	f_2	3	2	40

अथवा

एक परीक्षा में 100 विद्यार्थियों द्वारा प्राप्तांक नीचे दिये गए हैं।

प्राप्तांक	विद्यार्थियों की संख्या
0-5	2
5-10	5
10-15	6
15-20	8
20-25	10
25-30	25
30-35	20
35-40	18
40-45	4
45-50	2

एक 'से कम प्रकार का' संचयी बारंबारता वक्र खींचिए। अतः माध्यक ज्ञात कीजिए।

If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Or

The marks obtained by 100 students of a class in an examination are given below.

Marks	No. of Students
0-5	2
5-10	5
10-15	6
15-20	8
20-25	10
25-30	25
30-35	20
35-40	18
40-45	4
45-50	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.

Strictly Confidential: (For Internal and Restricted use only)
Secondary School Examination
March 2019
Marking Scheme – MATHEMATICS (SUBJECT CODE -041)

PAPER CODE: 30/1/1, 30/1/2, 30/1/3

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 1-80 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.

12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Let the point A be (x, y)

$\therefore \quad \frac{1+x}{2} = 2$ and $\frac{4+y}{2} = -3$ $\qquad \frac{1}{2}$

$\Rightarrow \quad x = 3$ and $y = -10$

$\therefore \quad$ Point A is (3, –10) $\qquad \frac{1}{2}$

2. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\qquad \frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\qquad \frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow \quad$ Product of roots = 1 $\qquad \frac{1}{2}$

$\Rightarrow \quad \frac{k}{3} = 1 \Rightarrow k = 3$ $\qquad \frac{1}{2}$

3. $\tan 2A = \cot (90° - 2A)$

$\therefore \quad 90° - 2A = A - 24°$ $\qquad \frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\qquad \frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\qquad \frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\qquad \frac{1}{2}$

4. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore$ 99 = 12 + (n − 1) × 3

$\Rightarrow$ n = 30 $\frac{1}{2}$

5. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore$ ar(ΔABC) : ar(ΔADE) = 9 : 1

6. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.) 1

e.g., 1.5, 1.6, 1.63 etc.

SECTION B

7. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

a = 4q + r, 0 ≤ r < 4

$\Rightarrow$ a = 4q, a = 4q + 1, a = 4q + 2 and a = 4q + 3. 1

Now a = 4q and a = 4q + 2 are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

a = 4q + 1 or a = 4q + 3 for some integer q. $\frac{1}{2}$

8. $a_n = a_{21} + 120$

$= (3 + 20 \times 12) + 120$

$= 363$ 1

$\therefore \quad 363 = 3 + (n - 1) \times 12$

$\Rightarrow \quad n = 31$ 1

or 31st term is 120 more than a_{21}.

OR

$a_1 = S_1 = 3 - 4 = -1$ $\frac{1}{2}$

$a_2 = S_2 - S_1 = [3(2)^2 - 4(2)] - (-1) = 5$ $\frac{1}{2}$

$\therefore\ d = a_2 - a_1 = 6$ $\frac{1}{2}$

Hence $a_n = -1 + (n - 1) \times 6 = 6n - 7$ $\frac{1}{2}$

Alternate method:

$S_n = 3n^2 - 4n$

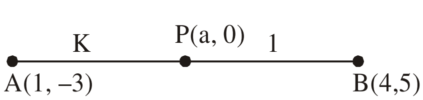
$\therefore \quad S_{n-1} = 3(n - 1)^2 - 4(n - 1) = 3n^2 - 10n + 7$ 1

Hence $a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - (3n^2 - 10n + 7)$ $\frac{1}{2}$

$= 6n - 7$ $\frac{1}{2}$

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$\therefore \quad a = \dfrac{4k+1}{k+1}$

$0 = \dfrac{5k-3}{k+1}$

$\Rightarrow \quad k = \dfrac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\dfrac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) = $1-\frac{1}{4}=\frac{3}{4}$. $\frac{1}{2}$

11. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$. 1

12. System of equations has infinitely many solutions

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$ $\frac{1}{2}$

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6 \text{ or } c = -6$ ...(1) $\frac{1}{2}$

Also $-3c = 3c - c^2 \Rightarrow c = 6 \text{ or } c = 0$ ...(2) $\frac{1}{2}$

From equations (1) and (2)

$c = 6.$ $\frac{1}{2}$

SECTION C

13. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and $b \neq 0$.

So $\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad a^2 = 2b^2$ 1

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2. $\frac{1}{2}$

Let a = 2 m for some integer m

$\therefore \quad b^2 = 2m^2$ $\frac{1}{2}$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2 $\frac{1}{2}$

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number. $\frac{1}{2}$

14. Sum of zeroes = k + 6 1

Product of zeroes = 2(2k – 1) 1

Hence $k + 6 = \frac{1}{2} \times 2(2k-1)$

$\Rightarrow \quad k = 7$ 1

15. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore \quad y = 3x$ ...(1) 1

and $y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get x = 7, y = 15.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

16. Let the required point on y-axis be (0, b) $\frac{1}{2}$

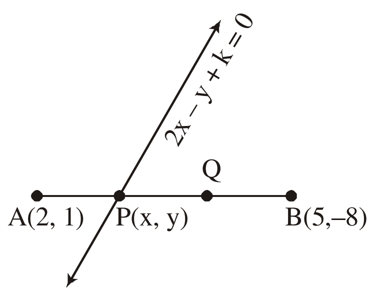
$\therefore \quad (5-0)^2 + (-2-b)^2 = (-3-0)^2 + (2-b)^2$ 1

$\Rightarrow \quad 29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow \quad b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2} + \frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on $2x - y + k = 0$

$\Rightarrow \quad 6 + 2 + k = 0$

$\Rightarrow \quad k = -8.$ 1

17. LHS $= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

$$\text{LHS} = \left(1 + \frac{1}{\tan A} - \text{cosec}\,A\right)(1 + \tan A + \sec A)$$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)\,(1 + \tan A + \sec A)$ 1

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$ 1

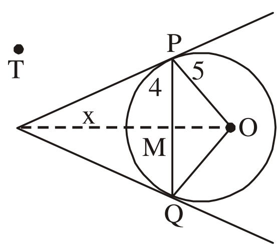
$$= (\sin A+\cos A-1)(\cos A+\sin A+1)\cdot\frac{1}{\cos A\sin A}$$

$$= \left[(\sin A+\cos A)^2-1\right]\times\frac{1}{\sin A\cos A}$$ 1

$$= (1+2\sin A\cos -1)\times\frac{1}{\sin A\cos A}$$ $\frac{1}{2}$

$= 2 = \text{RHS}$ $\frac{1}{2}$

18. Join OT and OQ. $\frac{1}{2}$



TP = TQ

$\therefore$ TM $\perp$ PQ and bisects PQ

Hence PM = 4 cm

Therefore OM = $\sqrt{25-16}=\sqrt{9}=3$ cm. $\frac{1}{2}$

Let TM = x

From ΔPMT, $PT^2 = x^2 + 16$

From ΔPOT, $PT^2 = (x + 3)^2 - 25$

Hence $x^2 + 16 = x^2 + 9 + 6x - 25$

$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$ 1

Hence $PT^2 = \frac{256}{9}+16=\frac{400}{9}$

$\therefore$ $PT=\frac{20}{3}$ cm. 1

19. ΔACB ~ ΔADC (AA similarity)

$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD}$ ...(1) 1

Also ΔACB ~ ΔCDB (AA similarity)

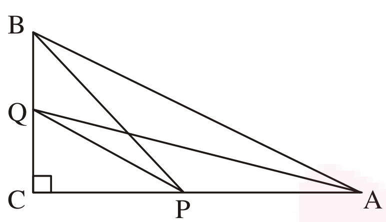
$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$\frac{AD}{CD} = \frac{CD}{BD}$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore \ AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2$. 1

20. $AC = \sqrt{64+36} = 10$ cm.

$\therefore$ Radius of the circle (r) = 5 cm. 1

Area of shaded region = Area of circle – Ar(ABCD) $\frac{1}{2}$

$= 3.14 \times 25 - 6 \times 8$ 1

$= 78.5 - 48$

$= 30.5 \text{ cm}^2$. $\frac{1}{2}$

21. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000 \text{ m}^3$ 1

If 8 cm standing water is needed

then area irrigated = $\dfrac{6\times1.5\times5000}{.08} = 562500\ m^2$. $1+\dfrac{1}{2}$

22. Modal class is 30-40 $\dfrac{1}{2}$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$

$= 30 + \left(\dfrac{16-10}{32-10-12}\right)\times 10$ 2

$= 36.$ $\dfrac{1}{2}$

SECTION D

23. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\dfrac{15}{8}$ hrs.

Therefore $\dfrac{1}{x} + \dfrac{1}{x-2} = \dfrac{8}{15}$ 2

$\Rightarrow 4x^2 - 23x + 15 = 0$ $\dfrac{1}{2}$

$\Rightarrow (4x - 3)(x - 5) = 0$

$x \neq \dfrac{3}{4} \quad \therefore x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\dfrac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\dfrac{30}{x-y} + \dfrac{44}{x+y} = 10$ ...(i) 1

and $\dfrac{40}{x-y} + \dfrac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get x = 8, y = 3. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

24. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$ 1

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ 1

Solving to get d = 2 $\frac{1}{2}$

and a = 7 $\frac{1}{2}$

$\therefore \quad Sn = \frac{n}{2}[14 + (n-1) \times 2]$

$= n(n + 6) \text{ or } (n^2 + 6n)$ 1

25. LHS = $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

Dividing num. & deno. by cos A

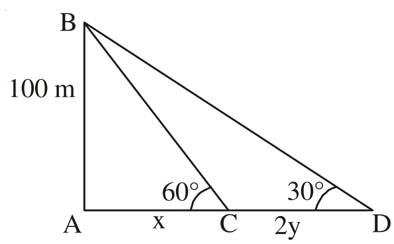
$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$ 1

$= \frac{\tan A - 1 + \sec A}{(\tan A - \sec A) + (\sec^2 A - \tan^2 A)}$ 1

$= \frac{\tan A - 1 + \sec A}{(\tan A - \sec A)(1 - \sec A - \tan A)}$ 1

$= \frac{-1}{\tan A - \sec A} = \frac{1}{\sec A - \tan A} = \text{RHS}$ 1

26. Correct Figure 1



Let the speed of the boat be y m/min

$\therefore$ CD = 2y

$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

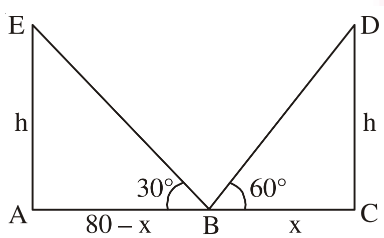
$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR

Correct Figure 1



Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80-x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

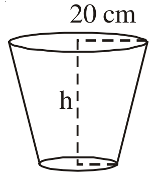
$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

27. Correct construction of ΔABC. 2

Correct construction of triangle similar to triangle ABC. 2

28.



Volume of the bucket = 12308.8 cm^3

Let $r_1 = 20$ cm, $r_2 = 12$ cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15 \text{ cm}$ 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l (r_1 + r_2)$

$= 3.14 (144 + 17 \times 32)$

$= 2160.32 \text{ cm}^2$. 1

29. Correct given, to prove, figure and construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

30.

Class	**Frequency**	**Cumulative freq.**
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$
30-40	12	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	40	

Correct Table 1

Median = 32.5 ⇒ median class is 30-40. $\frac{1}{2}$

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$ 1

$\Rightarrow f_1 = 3$ 1

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$ $\frac{1}{2}$

OR

Less than type distribution is as follows

Marks	No. of students		
Less than 5	2		
Less than 10	7		
Less than 15	13		
Less than 20	21		
Less than 25	31		
Less than 30	56		
Less than 35	76		
Less than 40	94		
Less than 45	98		
Less than 50	100	Correct Table	$1\frac{1}{2}$

Plotting of points (5, 2), (10, 7) (15, 13), (20, 21), (25, 31), (30, 56),

(35, 76), (40, 94), (45, 98), (50, 100) $1\frac{1}{2}$

Joining to get the curve $\frac{1}{2}$

Getting median from graph (approx. 29) $\frac{1}{2}$

QUESTION PAPER CODE 30/1/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Let the point A be (x, y)

$\therefore \quad \frac{x+3}{2} = -2$ and $\frac{y+4}{2} = 2$ $\frac{1}{2}$

$\Rightarrow \quad x = -7$ and $y = 0$

Point is (–7, 0) $\frac{1}{2}$

2. Any one rational number between $\sqrt{2}$ (1.41 approx.) and $\sqrt{3}$ (1.73 approx.) 1

e.g., 1.5, 1.6, 1.63 etc.

3. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore \quad 99 = 12 + (n - 1) \times 3$

$\Rightarrow \quad n = 30$ $\frac{1}{2}$

4. $\tan 2A = \cot (90° - 2A)$

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

5. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow$ $\frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

6. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore$ $\frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore$ $ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1$

SECTION B

7. System of equations has infinitely many solutions.

$\therefore$ $\frac{2}{k+1} = \frac{3}{2k-1} = \frac{7}{4k+1}$ $\frac{1}{2}$

$\Rightarrow$ $4k - 2 = 3k + 3$ $\frac{1}{2}$

$\Rightarrow$ $k = 5$

Also $12k + 3 = 14k - 7$ $\frac{1}{2}$

$\Rightarrow$ $k = 5$

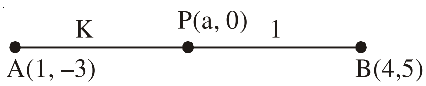
Hence $k = 5$. $\frac{1}{2}$

8. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) = $\frac{3}{6}$ or $\frac{1}{2}$. 1

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$\therefore \quad a = \frac{4k+1}{k+1}$

$0 = \frac{5k-3}{k+1}$

$\Rightarrow \quad k = \frac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\frac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) = $1 - \frac{1}{4} = \frac{3}{4}$. $\frac{1}{2}$

11. $a_n = a_{21} + 120$

$= (3 + 20 \times 12) + 120$

$= 363$ 1

$\therefore \quad 363 = 3 + (n-1) \times 12$

$\Rightarrow \quad n = 31$ 1

or 31st term is 120 more than a_{21}.

OR

$a_1 = S_1 = 3 - 4 = -1$ $\frac{1}{2}$

$a_2 = S_2 - S_1 = [3(2)^2 - 4(2)] - (-1) = 5$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 6$ $\frac{1}{2}$

Hence $a_n = -1 + (n-1) \times 6 = 6n - 7$ $\frac{1}{2}$

Alternate method:

$S_n = 3n^2 - 4n$

$\therefore \quad S_{n-1} = 3(n-1)^2 - 4(n-1) = 3n^2 - 10n + 7$ 1

Hence $a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - (3n^2 - 10n + 7)$ $\frac{1}{2}$

$= 6n - 7$ $\frac{1}{2}$

12. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r, \ 0 \leq r < 4$

$\Rightarrow \quad a = 4q, \ a = 4q + 1, \ a = 4q + 2$ and $a = 4q + 3.$ 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

SECTION C

13.

Class	x	Freq (f)	$u = \frac{x-50}{20}$	fu
0-20	10	12	–2	–24
20-40	30	15	–1	–15
40-60	50	32	0	0
60-80	70	k	1	k
80-100	90	13	2	26
		72 + k		–13 + k

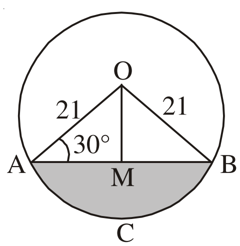
Correct Table 2

$\overline{x} = 53 = 50 + 20 \times \frac{-13+k}{72+k}$

$\Rightarrow$ 3k + 216 = 20k – 260

$\Rightarrow$ k = 28 1

14. Draw OM ⊥ AB



$\angle OAB = \angle OBA = 30°$ $\frac{1}{2}$

$\sin 30° = \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}$

$\cos 30° = \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21}{2}\sqrt{3}$

Area of ΔOAB $= \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

$= \frac{441}{4}\sqrt{3}\ \text{cm}^2.$ 1

∴ Area of shaded region = Area (sector OACB) – Area (ΔOAB)

$= \frac{22}{7} \times 21 \times 21 \times \frac{120}{360} - \frac{441}{4}\sqrt{3}$ 1

$= \left(462 - 441\frac{\sqrt{3}}{4}\right)\text{cm}^2$ or $271.3\ \text{cm}^2$ (approx.) $\frac{1}{2}$

15. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

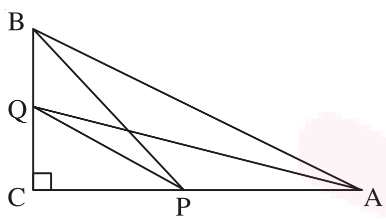
$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$\frac{AD}{CD} = \frac{CD}{BD}$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

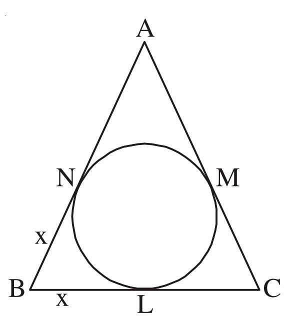
$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore \ AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2.$ 1

16.



Let BL = x = BN

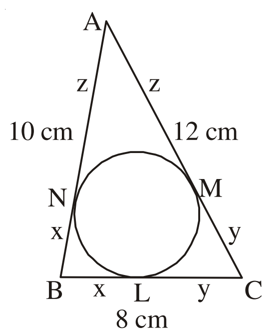
$\left.\begin{array}{l}\therefore \ CL = 8 - x = CM \\ \because \ AC = 12 \Rightarrow AM = 4 + x = AN\end{array}\right\}$ 1

Now AB = AN + NB = 10 $\Rightarrow$ x + 4 + x = 10

$\Rightarrow$ x = 3 1

$\therefore$ BL = 3 cm, CM = 5 cm and AN = 7 cm 1

Alternate method



Let BL = BN = x (tangents from external points are equal) $\frac{1}{2}$

CL = CM = y

AN = AM = z

$\therefore$ AB + BC + AC = 2x + 2y + 2z = 30

$\Rightarrow$ x + y + z = 15 ...(i) 1

Also x + z = 10, x + y = 8 and y + z = 12

Subtracting from equation (i)

y = 5, z = 7 and x = 3 $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

$\therefore$ BL = 3 cm, CM = 5 cm and AN = 7 cm.

17. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000$ m^3 1

If 8 cm standing water is needed

then area irrigated = $\frac{6\times1.5\times5000}{.08} = 562500\ m^2$. $1+\frac{1}{2}$

18. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and b ≠ 0.

So $\sqrt{2} = \frac{a}{b}$

$\Rightarrow$ $a^2 = 2b^2$ 1

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2. $\frac{1}{2}$

Let a = 2 m for some integer m

$\therefore$ $b^2 = 2m^2$ $\frac{1}{2}$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2 $\frac{1}{2}$

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number. $\frac{1}{2}$

19. Sum of zeroes = k + 6 1

Product of zeroes = 2(2k – 1) 1

Hence k + 6 = $\frac{1}{2} \times 2(2k-1)$

$\Rightarrow$ k = 7 1

20. Let the required point on y-axis be (0, b) $\frac{1}{2}$

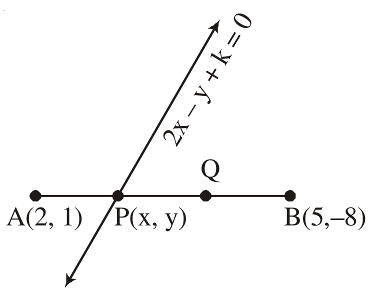
$\therefore$ $(5-0)^2 + (-2-b)^2 = (-3-0)^2 + (2-b)^2$ 1

$\Rightarrow$ $29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow$ b = –2 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2} + \frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on 2x – y + k = 0

$\Rightarrow$ 6 + 2 + k = 0

$\Rightarrow$ k = –8. 1

21. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore$ y = 3x ...(1) 1

and y + 5 = 2(x + 10) ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get $x = 7$, $y = 15$.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

22. LHS $= \sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

LHS $= \left(1 + \frac{1}{\tan A} - \text{cosec}\,A\right)(1 + \tan A + \sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)\,(1 + \tan A + \sec A)$ 1

$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1 + \tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$ 1

$$= (\sin A+\cos A-1)(\cos A+\sin A+1)\cdot\frac{1}{\cos A\sin A}$$

$$= \left[(\sin A+\cos A)^2-1\right]\times\frac{1}{\sin A\cos A}$$ 1

$$= (1+2\sin A\cos -1)\times\frac{1}{\sin A\cos A}$$ $\frac{1}{2}$

$$= 2 = \text{RHS}$$ $\frac{1}{2}$

SECTION D

23. $$\text{LHS} = \frac{\sin^2 A/\cos^2 A}{\dfrac{\sin^2 A}{\cos^2 A}-1}+\frac{1/\sin^2 A}{\dfrac{1}{\cos^2 A}-\dfrac{1}{\sin^2 A}}$$ 1

$$= \frac{\sin^2 A}{\sin^2 A-\cos^2 A}+\frac{\cos^2 A}{\sin^2 A-\cos^2 A}$$ 1

$$= \frac{1}{\sin^2 A-\cos^2 A}$$ 1

$$= \frac{1}{1-2\cos^2 A}$$ 1

24. Here $a = 3$, $a_n = 83$ and $S_n = 903$

Therefore $83 = 3 + (n - 1)d$

$\Rightarrow$ $(n - 1)d = 80$ ...(i) 1

Also $903 = \frac{n}{2}[2a+(n-1)d]=\frac{n}{2}(6+80)=43n$ (using (i)) $1+\frac{1}{2}$

$\Rightarrow$ $n = 21$

and $d = 4$ $1\frac{1}{2}$

25. Correct construction of ΔABC — 2

Correct construction of triangle similar to ΔABC. — 2

26.

Class	**Frequency**	**Cumulative freq.**		
0-10	f_1	f_1		
10-20	5	$5 + f_1$		
20-30	9	$14 + f_1$		
30-40	12	$26 + f_1$		
40-50	f_2	$26 + f_1 + f_2$		
50-60	3	$29 + f_1 + f_2$		
60-70	2	$31 + f_1 + f_2$	Correct Table	1
	40			

Median = 32.5 $\Rightarrow$ median class is 30-40. — $\frac{1}{2}$

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$ — 1

$\Rightarrow f_1 = 3$ — 1

Also $31 + f_1 + f_2 = 40$

$\Rightarrow f_2 = 6$ — $\frac{1}{2}$

OR

Less than type distribution is as follows

Marks	No. of students		
Less than 5	2		
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Less than 25	31		
Less than 30	56		
Less than 35	76		
Less than 40	94		
Less than 45	98		
Less than 50	100	Correct Table	$1\frac{1}{2}$

Plotting of points (5, 2), (10, 7) (15, 13), (20, 21), (25, 31), (30, 56),

(35, 76), (40, 94), (45, 98), (50, 100) $1\frac{1}{2}$

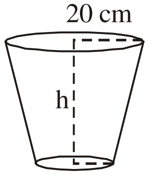
Joining to get the curve $\frac{1}{2}$

Getting median from graph (approx. 29) $\frac{1}{2}$

27. Correct given, to prove, figure and construction $\frac{1}{2}\times 4=2$

Correct proof. 2

28.



Volume of the bucket = $12308.8\ cm^3$

Let $r_1 = 20$ cm, $r_2 = 12$ cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow h = \frac{12308.8 \times 3}{3.14 \times 784} = 15$ cm 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l\,(r_1 + r_2)$

$= 3.14\ (144 + 17 \times 32)$

$= 2160.32\ cm^2$. 1

29. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow \quad 4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow \quad (4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4} \quad \therefore\ x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

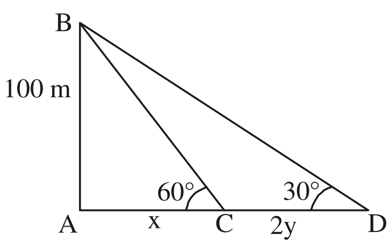
$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get x = 8, y = 3. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

30. Correct Figure 1



Let the speed of the boat be y m/min

$\therefore\ CD = 2y$

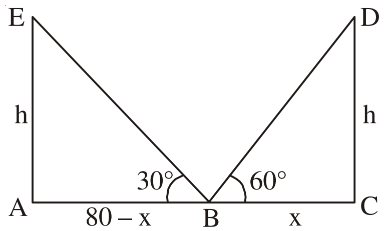
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore\ y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR



Correct Figure 1

Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80 - x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

QUESTION PAPER CODE 30/1/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. LCM $(x^3y^2, xy^3) = x^3y^3$. 1

2. Numbers are 12, 15, 18, ..., 99 $\frac{1}{2}$

$\therefore \quad 99 = 12 + (n - 1) \times 3$

$\Rightarrow \quad n = 30$ $\frac{1}{2}$

3. AB = 1 + 2 = 3 cm $\frac{1}{2}$

$\Delta ABC \sim \Delta ADE$

$\therefore \quad \frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$ $\frac{1}{2}$

$\therefore \quad ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1$

4. Let the point A be (x, y)

$\therefore \quad \frac{1+x}{2} = 2$ and $\frac{4+y}{2} = -3$ $\frac{1}{2}$

$\Rightarrow \quad x = 3$ and $y = -10$

$\therefore \quad$ Point A is (3, –10) $\frac{1}{2}$

5. Since roots of the equation $x^2 + 4x + k = 0$ are real

$\Rightarrow \quad 16 - 4k \geq 0$ $\frac{1}{2}$

$\Rightarrow \quad k \leq 4$ $\frac{1}{2}$

OR

Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other

$\Rightarrow \quad$ Product of roots = 1 $\frac{1}{2}$

$\Rightarrow \quad \frac{k}{3} = 1 \Rightarrow k = 3$ $\frac{1}{2}$

6. $\tan 2A = \cot(90° - 2A)$

$\therefore \quad 90° - 2A = A - 24°$ $\frac{1}{2}$

$\Rightarrow \quad A = 38°$ $\frac{1}{2}$

OR

$\sin 33° = \cos 57°$ $\frac{1}{2}$

$\therefore \quad \sin^2 33° + \sin^2 57° = \cos^2 57° + \sin^2 57° = 1$ $\frac{1}{2}$

SECTION B

7. Required numbers are

$14, 21, 28, 35, ..., 98.$ 1

$98 = 14 + (n - 1) \times 7$ $\frac{1}{2}$

$\Rightarrow \quad n = 13$ $\frac{1}{2}$

OR

Given $S_n = n^2$

$S_1 = a_1 = 1$ $\frac{1}{2}$

$S_2 = a_1 + a_2 = 4$

$\Rightarrow a_2 = 3$ $\frac{1}{2}$

$\therefore d = a_2 - a_1 = 2$ $\frac{1}{2}$

$a_{10} = 1 + 18 = 19$ $\frac{1}{2}$

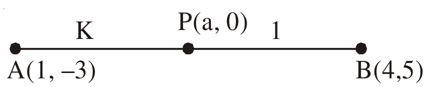
8. Total number of outcomes = 8 $\frac{1}{2}$

Favourable number of outcomes (HHH, TTT) = 2 $\frac{1}{2}$

Prob. (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ $\frac{1}{2}$

$\therefore$ Prob. (losing the game) $= 1 - \frac{1}{4} = \frac{3}{4}$. $\frac{1}{2}$

9. Let the required point be (a, 0) and required ratio AP : PB = k : 1 $\frac{1}{2}$



$$\therefore \quad a = \frac{4k+1}{k+1}$$

$$0 = \frac{5k-3}{k+1}$$

$\Rightarrow$ $k = \frac{3}{5}$ or required ratio is 3 : 5 1

Point P is $\left(\frac{17}{8}, 0\right)$ $\frac{1}{2}$

10. Total number of outcomes = 6.

(i) Prob. (getting a prime number (2, 3, 5)) $= \frac{3}{6}$ or $\frac{1}{2}$ 1

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)) $= \frac{3}{6}$ or $\frac{1}{2}$. 1

11. System of equations has infinitely many solutions

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$ $\frac{1}{2}$

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6$ or $c = -6$ ...(1) $\frac{1}{2}$

Also $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ ...(2) $\frac{1}{2}$

From equations (1) and (2)

$c = 6$. $\frac{1}{2}$

12. Using Euclid's Algorithm

$$\left.\begin{aligned} 7344 &= 1260 \times 5 + 1044 \\ 1260 &= 1044 \times 1 + 216 \\ 1044 &= 216 \times 4 + 180 \\ 216 &= 180 \times 1 + 36 \\ 180 &= 36 \times 5 + 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

HCF of 1260 and 7344 is 36. $\frac{1}{2}$

OR

Using Euclid's Algorithm

$a = 4q + r, 0 \le r < 4$

$\Rightarrow$ $a = 4q, a = 4q + 1, a = 4q + 2$ and $a = 4q + 3$. 1

Now $a = 4q$ and $a = 4q + 2$ are even numbers. $\frac{1}{2}$

Therefore when a is odd, it is of the form

$a = 4q + 1$ or $a = 4q + 3$ for some integer q. $\frac{1}{2}$

SECTION C

13. Let $p(x) = 3x^3 + 10x^2 - 9x - 4$.

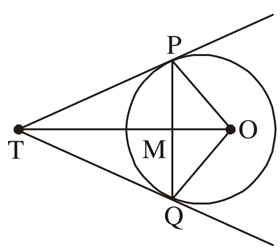
One of the zeroes is 1, therefore dividing p(x) by (x – 1)

$p(x) = (x - 1)(3x^2 + 13x + 4)$ $1\frac{1}{2}$

$= (x - 1)(x + 4)(3x + 1)$ 1

All zeroes are $x = 1, x = -4$ and $x = -\frac{1}{3}$. $\frac{1}{2}$

14. Join OQ, TP = TQ $\therefore$ TM $\perp$ PQ and bisects PQ



Hence PM = 4 cm.

$\therefore$ OM = $\sqrt{25-16} = \sqrt{9} = 3\,\text{cm}$ 1

Let TM = x $\therefore$ $PT^2 = x^2 + 16$ (ΔPMT)

$PT^2 = (x + 3)^2 - 25$ (ΔPOT)

Hence $x^2 + 16 = (x + 3)^2 - 25 = x^2 + 9 + 6x - 25$ 1

$\Rightarrow 6x = 32 \Rightarrow x = \frac{16}{3}$

Hence $PT^2 = \frac{256}{9} + 16 = \frac{400}{9}$

$\Rightarrow PT = \frac{20}{3}\text{cm}$ 1

15. Let us assume $\frac{2+\sqrt{3}}{5}$ be a rational number.

Let $\frac{2+\sqrt{3}}{5}=\frac{a}{b}$ (b ≠ 0, a and b are integers)

$\Rightarrow \quad \sqrt{3} = \frac{5a-2b}{b}$ 1

∵ a, b are integers

∴ $\frac{5a-2b}{b}$ is a rational number 1

i.e. $\sqrt{3}$ is a rational number

which contradicts the fact that $\sqrt{3}$ is irrational

Therefore is $\frac{2+\sqrt{3}}{5}$ is an irrational number. 1

16. LHS = $\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$ 1

$= (\sin^2\theta + \cos^2\theta) + \text{cosec}^2\theta + \sec^2\theta + \frac{2\sin\theta}{\sin\theta} + 2\frac{\cos\theta}{\cos\theta}.$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$ $1\frac{1}{2}$

$= 7 + \cot^2\theta + \tan^2\theta = \text{RHS}$ $\frac{1}{2}$

OR

LHS $= \left(1+\frac{1}{\tan A} - \text{cosec}\,A\right)(1+\tan A + \sec A)$

$= \frac{1}{\tan A}(\tan A + 1 - \sec A)\,(1+\tan A+\sec A)$ 1

$= \frac{1}{\tan A}[(1+\tan A)^2 - \sec^2 A]$ 1

$= \frac{1}{\tan A}[1+\tan^2 A + 2\tan A - 1 - \tan^2 A]$

$= 2 = \text{RHS}$ 1

Alternate method

$$\text{LHS} = \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right) \quad 1$$

$$= (\sin A + \cos A - 1)(\cos A + \sin A + 1)\cdot\frac{1}{\cos A \sin A}$$

$$= \left[(\sin A + \cos A)^2 - 1\right]\times\frac{1}{\sin A \cos A} \quad 1$$

$$= (1 + 2\sin A \cos - 1)\times\frac{1}{\sin A \cos A} \quad \frac{1}{2}$$

$$= 2 = \text{RHS} \quad \frac{1}{2}$$

17. Let sum of the ages of two children be x yrs and father's age be y yrs.

$\therefore$ $y = 3x$ ...(1) 1

and $y + 5 = 2(x + 10)$ ...(2) 1

Solving equations (1) and (2)

$x = 15$

and $y = 45$

Father's present age is 45 years. 1

OR

Let the fraction be $\frac{x}{y}$

$\therefore$ $\frac{x-2}{y} = \frac{1}{3}$ ...(1) 1

and $\frac{x}{y-1} = \frac{1}{2}$ ...(2) 1

Solving (1) and (2) to get $x = 7$, $y = 15$.

$\therefore$ Required fraction is $\frac{7}{15}$ 1

18. Let the required point on y-axis be (0, b) $\frac{1}{2}$

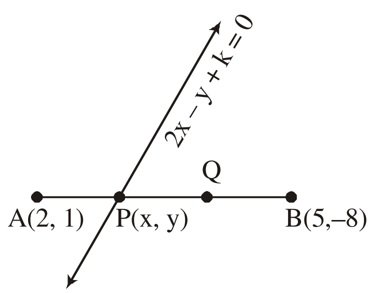
$\therefore$ $(5 - 0)^2 + (-2 - b)^2 = (-3 - 0)^2 + (2 - b)^2$ 1

$\Rightarrow$ $29 + 4b + b^2 = 13 + b^2 - 4b$

$\Rightarrow$ $b = -2$ 1

$\therefore$ Required point is (0, –2) $\frac{1}{2}$

OR



AP : PB = 1 : 2 $\frac{1}{2}$

$x = \frac{4+5}{3} = 3$ and $y = \frac{2-8}{3} = -2$ $\frac{1}{2}+\frac{1}{2}$

Thus point P is (3, –2). $\frac{1}{2}$

Point (3, –2) lies on $2x - y + k = 0$

$\Rightarrow$ $6 + 2 + k = 0$

$\Rightarrow$ $k = -8$. 1

19. Modal class is 30-40 $\frac{1}{2}$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 30 + \left(\frac{16 - 10}{32 - 10 - 12}\right) \times 10$ 2

$= 36$. $\frac{1}{2}$

20. Length of canal covered in 30 min = 5000 m. $\frac{1}{2}$

$\therefore$ Volume of water flown in 30 min = $6 \times 1.5 \times 5000\ m^3$ 1

If 8 cm standing water is needed

then area irrigated = $\frac{6 \times 1.5 \times 5000}{.08} = 562500\ m^2$. $1+\frac{1}{2}$

21. $\Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD}$ ...(1) 1

Also $\Delta ACB \sim \Delta CDB$ (AA similarity)

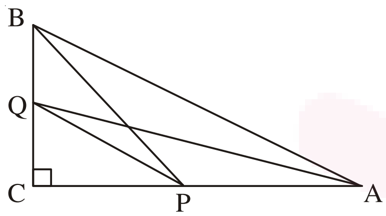
$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD}$ ...(2) 1

Using equations (1) and (2)

$\frac{AD}{CD} = \frac{CD}{BD}$

$\Rightarrow \quad CD^2 = AD \times BD$ 1

OR



Correct Figure $\frac{1}{2}$

$AQ^2 = CQ^2 + AC^2$ 1

$BP^2 = CP^2 + BC^2$ $\frac{1}{2}$

$\therefore AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2$. 1

22. $AC = \sqrt{64+36} = 10$ cm.

$\therefore$ Radius of the circle (r) = 5 cm. 1

Area of shaded region = Area of circle – Ar(ABCD) $\frac{1}{2}$

$= 3.14 \times 25 - 6 \times 8$ 1

$= 78.5 - 48$

$= 30.5$ cm^2. $\frac{1}{2}$

SECTION D

23. $\sec^2\theta = \left(x + \frac{1}{4x}\right)^2 = x^2 + \frac{1}{16x^2} + \frac{1}{2}$ 1

$\therefore \quad \tan^2\theta = \sec^2\theta - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$ 1

$\Rightarrow \quad \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \tan\theta = \left(x - \frac{1}{4x}\right)$ or $\left(\frac{1}{4x} - x\right)$ 1

Hence $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$ 1

24. Correct given, to prove, figure, construction $\frac{1}{2} \times 4 = 2$

Correct proof. 2

25. Less than type distribution is as follows

Daily income	Number of workers
Less than 220	12
Less than 240	26
Less than 260	34
Less than 280	40
Less than 300	50

Correct Table $1\frac{1}{2}$

Plotting of points (220, 12), (240, 26), (260, 34) (280, 40) and (300, 50) $1\frac{1}{2}$

Joining to get curve 1

OR

Daily expenditure	x_i	No. of households (f_i)	$u_i = \frac{x - 225}{50}$	$f_i u_i$
100-150	125	4	–2	–8
150-200	175	5	–1	–5
200–250	225	12	0	0
250–300	275	2	1	2
300-350	325	2	2	4
		$\Sigma f_i = 25$		$\Sigma f_i u_i = -7$

Correct Table 2

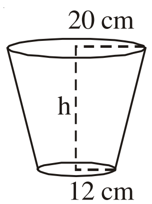
Mean = $225 + 50 \times \left(\frac{-7}{25}\right) = 211$ 2

Mean expenditure on food is ₹ 211.

26. Correct construction of ΔABC. 2

Correct construction of triangle similar to triangle ABC. 2

27.



Volume of the bucket = 12308.8 cm^3

Let r_1 = 20 cm, r_2 = 12 cm

$\therefore \quad V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$

$\therefore \quad 12308.8 = \frac{3.14 \times h}{3}(400 + 144 + 240)$ 1

$\Rightarrow \quad h = \frac{12308.8 \times 3}{3.14 \times 784} = 15$ cm 1

Now $l^2 = h^2 + (r_1 - r_2)^2 = 225 + 64 = 289$

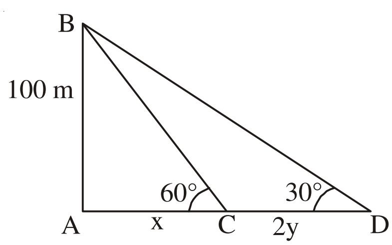
$\Rightarrow \quad l = 17$ cm. 1

Surface area of metal sheet used = $\pi r_2^2 + \pi l\,(r_1 + r_2)$

= 3.14 (144 + 17 × 32)

= 2160.32 cm^2. 1

28.



Correct Figure 1

Let the speed of the boat be y m/min

$\therefore$ CD = 2y

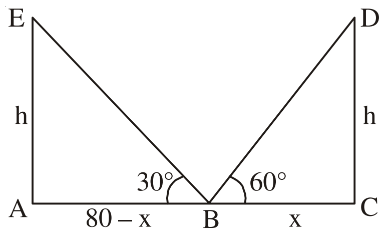
$\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ 1

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x + 2y} \Rightarrow x + 2y = 100\sqrt{3}$ 1

$\therefore \; y = \frac{100\sqrt{3}}{3} = 57.73$ 1

or speed of boat = 57.73 m/min.

OR



Correct Figure 1

Let BC = x so AB = 80 – x

where AC is the road.

$\tan 60° = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$ 1

and $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80 - x} \Rightarrow h\sqrt{3} = 80 - x$ 1

Solving equation to get

$x = 20,\ h = 20\sqrt{3}$

$\therefore$ AB = 60 m, BC = 20 m and h = $20\sqrt{3}$ m. 1

29. Let the smaller tap fills the tank in x hrs

$\therefore$ the larger tap fills the tank in (x – 2) hrs.

Time taken by both the taps together = $\frac{15}{8}$ hrs.

Therefore $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ 2

$\Rightarrow 4x^2 - 23x + 15 = 0$ $\frac{1}{2}$

$\Rightarrow (4x - 3)(x - 5) = 0$

$x \neq \frac{3}{4} \quad \therefore x = 5$ 1

Smaller and larger taps can fill the tank seperately in 5 hrs and 3 hrs resp. $\frac{1}{2}$

OR

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Given $\frac{30}{x-y} + \frac{44}{x+y} = 10$ ...(i) 1

and $\frac{40}{x-y} + \frac{55}{x+y} = 13$ ...(ii) 1

Solving (i) and (ii) to get

$x + y = 11$ ..(iii)

and $x - y = 5$ ...(iv)

Solving (iii) and (iv) to get $x = 8$, $y = 3$. 1+1

Speed of boat = 8 km/hr & speed of stream = 3 km/hr.

30. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$ 1

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$ 1

Solving to get $d = 2$ $\frac{1}{2}$

and $a = 7$ $\frac{1}{2}$

$\therefore \quad Sn = \frac{n}{2}[14 + (n-1) \times 2]$

$= n(n + 6)$ or $(n^2 + 6n)$ 1